Numerical Simulation of the Diamond Grit Friability Tester

J. Spinnewyn, R. Lebout and R. Lorent

Custodiam s.a., 111 Rue Egide Van Ophemstraat, B-1180 Brussels, Belgium

ABSTRACT:
Synthetic diamond grit producers and users developed several methods to measure the impact strength of the grits they produce or use. A widespread approach, also adopted by the FEPA, consists of putting a few carats of diamond grits in a capsule, together with a steel ball, and of shaking it a number of times in order to subsequently measure the damage caused to the grits. Such tests are often used blindly, without wondering about the physics behind.
For a better understanding of the motion of the steel ball in the capsule and of the effect of variations in the machine parameters, we have developed a simple model simulating this motion, with as main output the energy transfer from the ball to the diamond grits. The equations are solved numerically. The results are a nice example of the fact that, even in such a simple dynamic system, chaotic motion is possible.

1. The mathematical model

For the simulation of the comminution apparatus, as described in the FEPA standard [1] for testing relative strengths of saw diamond grits, we have simplified the model:

- by assuming that the **capsule motion** is a harmonic oscillation along a **straight line.**
- by **neglecting** the **transversal modes** in the motion of the steel ball.
- by assuming that both **capsule ends** are **flat**.
- the diamond grits are not taken into account individually but as a layer**,** present at each capsule end at impact of the steel ball.

This simplified geometry (see fig. 1) allows us to use formulae for direct central collisions.
Taking into account that the mass of the steel ball is much lighter than the capsule mass, we obtain the following equations :

- for the ball velocity after collision at one of the capsule ends:

$$v_{ba} = (1+k) * v_{cb} - k * v_{bb} \qquad (1)$$

- for the container velocity after collision

$$v_{ca} = v_{cb} \qquad (2)$$

with : v_{ba}: ball speed after collision with capsule end
v_{bb}: ball speed before collision with capsule end
v_{ca}: capsule speed after collision with the steel ball.
v_{cb}: capsule speed before collision with the steel ball.
k: coefficient of restitution, = the absolute value of $(v_{ca}- v_{ba})/ (v_{bb}- v_{cb})$
with $0 \leq k \leq 1$
when k = 0 we have a perfectly inelastic collision
k = 1 we have a perfectly elastic collision

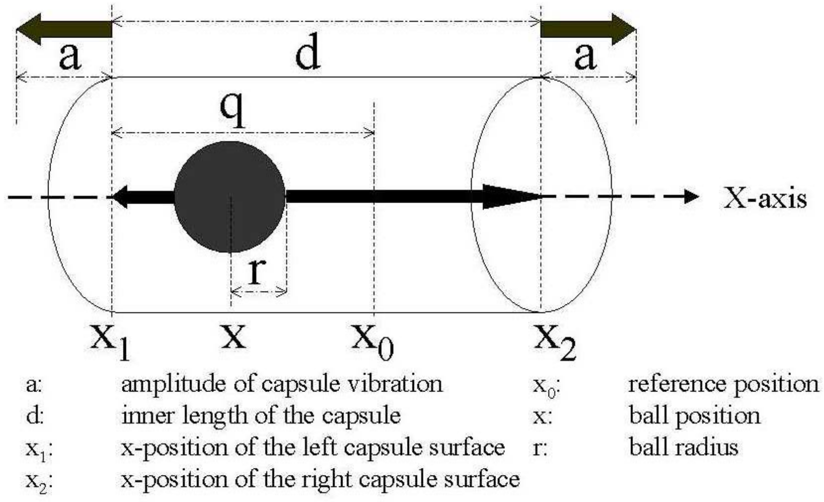

Fig. 1: geometry used in the simulation and definition of the symbols

The decrease in the ball's kinetic energy caused by collision is given by:

$$-\Delta E = 0.5 * m_b * (v_{cb} - v_{bb})^2 * (1 - k^2) \tag{3}$$

with m_b: mass of the steel ball

Equation 3 confirms that, for a given k-value, the energy transfer reaches a maximum if v_{cb} and v_{bb} have opposite signs (i.e. move into opposite directions) and maximum absolute values.

Furthermore we have :

$$x_0(t) = a * \cos(\omega t) \tag{4}$$
$$x_1(t) = x_0(t) - q \tag{5}$$
$$x_2(t) = x_1(t) + d \tag{6}$$

With: x_0, x_1, x_2 and q, defined in fig. 1.
t = time
$\omega = 2 * \pi * f$
f = frequency of the capsule vibration

The motion of the steel ball between two consecutive collisions is ballistic. This means that, between two consecutive collisions, respectively at $t = t_{n-1}$ and $t = t_n$, the equation of motion of the steel ball centre will be given by equation 7:

$$x(t) = x(t_{n-1}) + v_{ba} * (t - t_{n-1}) \tag{7}$$

The steel ball is colliding with one of the capsule ends when its centre, x, obeys one of the following equations :

$$x(t) = x_1(t) + r + s \text{ (left capsule end, see fig. 1)} \tag{8}$$
$$x(t) = x_2(t) - r - s \text{ (right capsule end, see fig. 1)} \tag{9}$$

with: s: thickness of the layer of diamonds at each capsule end.

Because of the existence of non-periodic solutions, we have opted for a numerical approach to find the consecutive collisions of the steel ball with each of the capsule ends.
We have developed a programme, which uses the root finding function RTSAFE from [2] (combination of the bisection and the Newton-Raphson algorithm) to solve equations (8) and (9).
At each collision (whatever the capsule end) we also calculate the energy transfer from the ball to the diamond layer using equation 3.
At the end of the simulation (typically 400 collisions after having reached a stable regime), the programme also calculates the energy transfer rate <dE/dt> , i.e. the average energy transfer per second from the steel ball to the diamond layers present at each capsule end.

2. The parameters

Unless mentioned otherwise, our simulation uses the following default parameter values:

Values as imposed by the FEPA standard [1]:

f:	capsule frequency	40 Hz
a:	amplitude of capsule vibration	4.05 mm
d:	inner capsule length	22.94 mm
r:	steel ball radius	3.97 mm
m_b:	steel ball mass	2.042 g

The thickness of the diamond layer present at each of the capsule ends at the moment of collision can not be measured. For coarser grits, the average diamond grit diameter is a good guess for it, at least at the beginning of the test. Therefore, we have opted for:
$s = 600\ \mu m$
For the coefficient of restitution (see next paragraph) we have chosen the following default value:
$k = 0.1$

To test the effect of a parameter, we vary it keeping all the others equal to their default value.

3. Results of the simulation

3.1 The effect of k

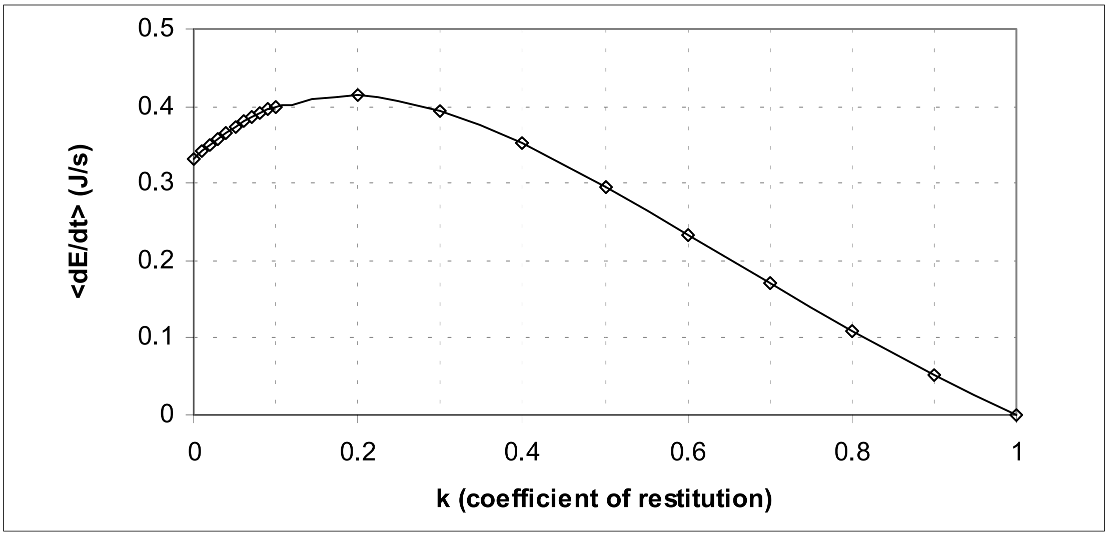

Fig. 2: effect of k on energy transfer rate (all other parameters equal to the default values of §2)

With all other parameters equal to the reference values of §2, the computer programme points out the existence of a maximum energy transfer rate at about k = 0.2. (fig. 2)
Observations of a steel ball bouncing on a flat capsule end covered with a layer of diamond grits indicate that k values are mostly in the range $0 \leq k \leq 0.3$.

- k = 0 corresponds (see equation 3) to a complete standstill of the ball after collision. We take this as the lower limit in our simulations because, in case of flowerlike-debris, as obtained after comminution tests lasting much longer than the half life, the steel ball does not bounce back noticeably.
- k = 0.3 corresponds to a ball bouncing back to about 1/10th of the height from where it was dropped. This seems to be a maximum for unbroken grit (typical for the situation at the beginning of the comminution test).

3.2 The effect of the inner capsule length d

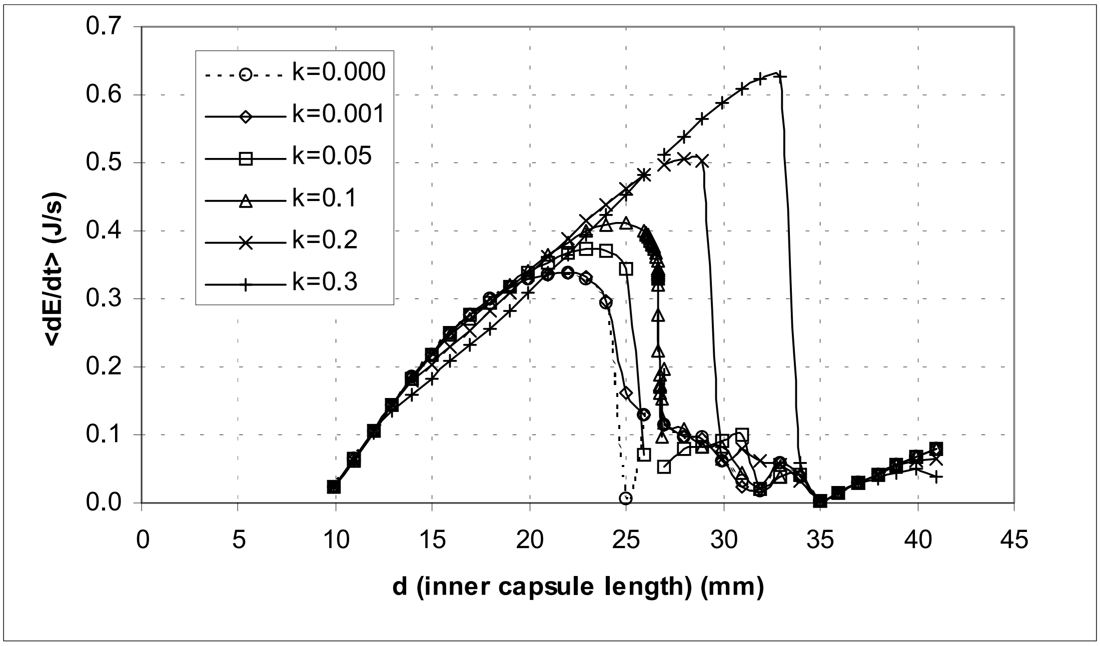

Fig. 3: effect of d on the energy transfer rate for different k (all other parameters as in §2)

For each coefficient of restitution, the dependence on the inner capsule length is more or less the same: starting at the shortest d, the energy transfer rate increases gradually towards an absolute maximum and then drops steeply to very low values. Further increase of d still gives a number of local maxima, which stay far below the absolute maximum.
The **absolute maximum** in the energy transfer rate is reached (see equation 3) **when** the steel ball hits the **returning capsule end** at the moment the latter is **at maximum speed** (which occurs at zero deviation). The ball motion in such case is shown in fig. 4 (case k=0.1, d=22.94 mm).
For **shorter d**, the opposite face is hit before it reaches maximum speed, which explains the decrease in energy transfer rate. At much shorter d, e.g. d=15.94 mm, v_{ba} is so low that the steel ball will be hit once more by the same capsule end before reaching the other side. (see fig. 4)
For **d values beyond the absolute maximum**, the steel ball will hit the opposite face after the latter reaches for the first time its maximum speed in the opposite direction. The ball **motion** strongly **depends on** the **phase angle of the collision**. The motion can be **chaotic** (illustrated for d=32.94 mm in fig. 4) or **repetitive** (at a frequency lower than f) as illustrated for d=40.92 mm.
Moreover, fig. 3 clearly shows that the **lower k**, the **shorter** becomes the **inner capsule length at** which the **maximum** occurs. This is because lower k means lower ball velocity and, hence, shorter distances the ball can cover before meeting the opposite face at the right time.
Fig. 3. also shows that the inner capsule length of about **23 mm**, as **imposed by the FEPA**, appears to be the highest value one can take without taking the risk of falling off the top of the energy maximum. This result confirms that the model pretty well simulates the longitudinal motion.

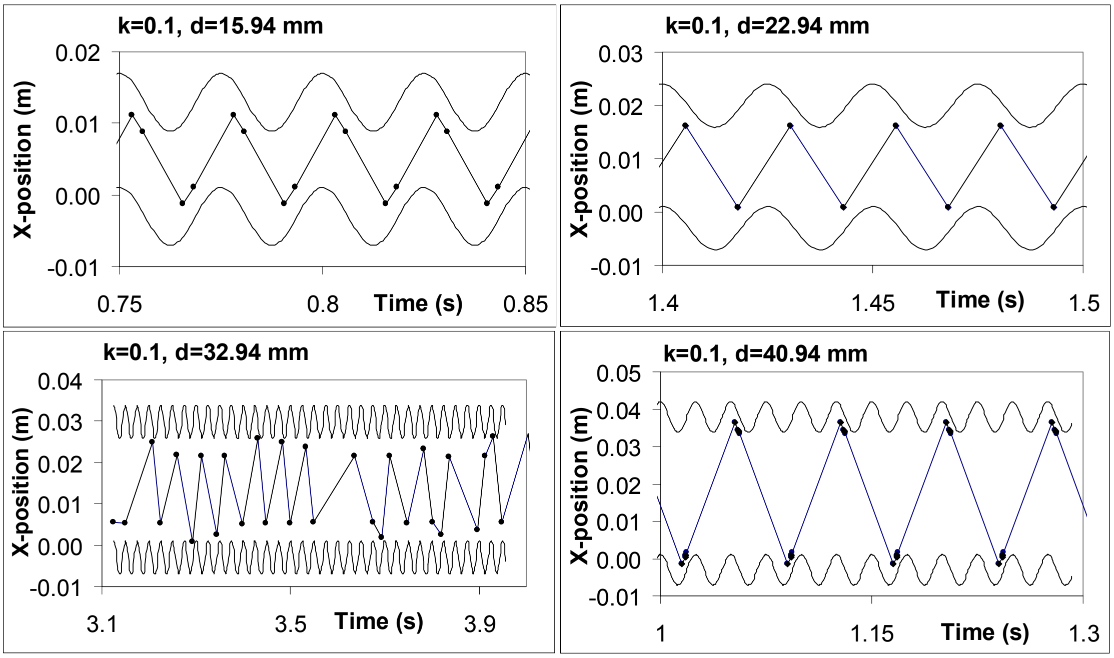

Fig. 4: motions for selected points of fig. 2 (sawtooth: ball centre; 2 cosines: capsule ends)

3.3 The effect of the amplitude of the capsule vibration

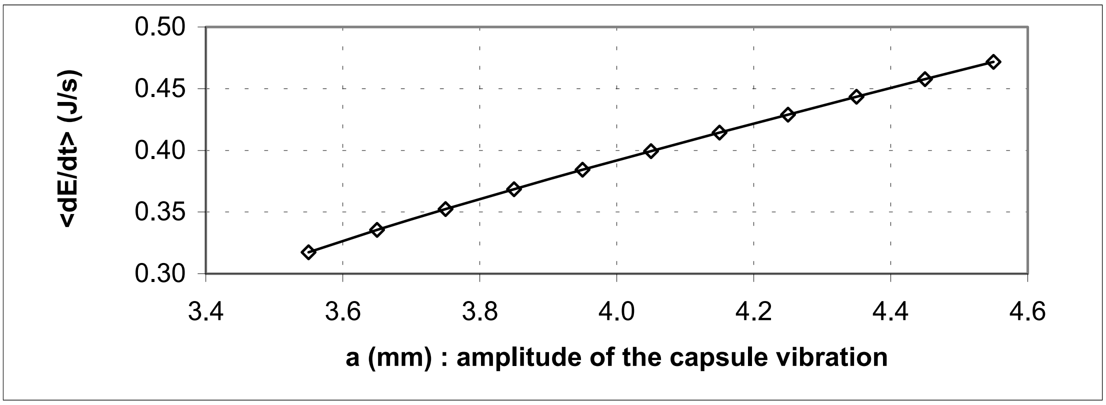

Fig. 5: effect of the amplitude of the capsule vibration (all other parameters as in §2)

3.4 The effect of the frequency of the capsule vibration

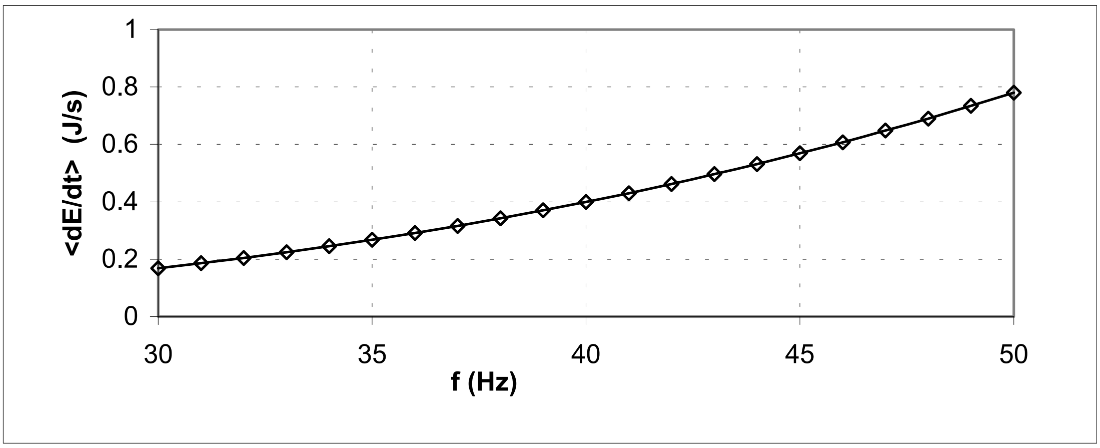

Fig. 6: effect of capsule frequency (all other parameters as in §2)

3.5 The effect of the steel ball size

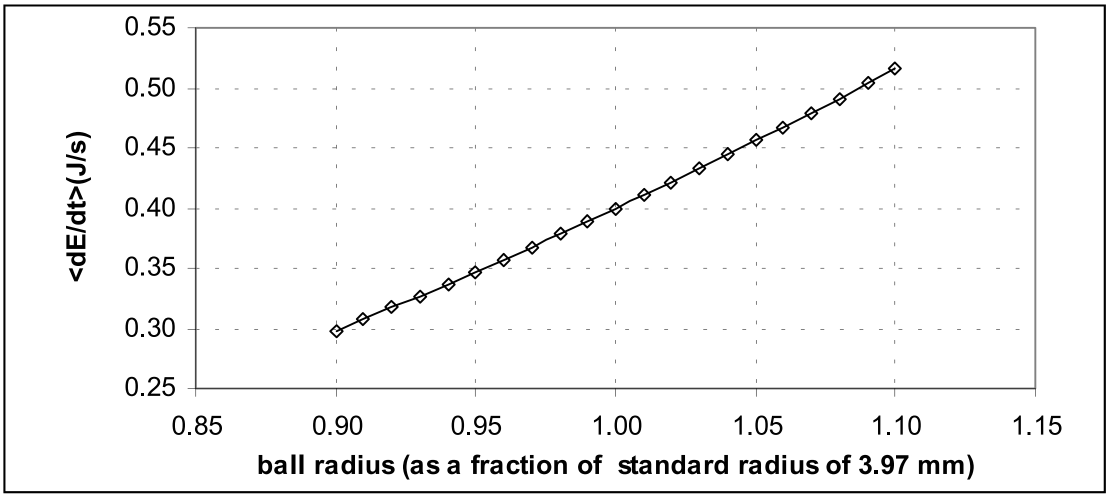

Fig. 7: effect of (relative) steel ball size (and mass which varies with r^3)
(all other parameters as in §2)

4. Conclusions

We have developed a simple mathematical model in order to simulate the comminution apparatus, as described in the FEPA standard [1] for measuring the Impact Strength of saw diamond grits. Our model numerically simulates the longitudinal motion of the steel ball in the capsule. It also calculates the energy transfer rate of the steel ball to the layer of diamond grits present at each capsule end at the collision.

The fact that the model explains why the inner capsule length imposed by the FEPA standard is about 23 mm, confirms that it simulates pretty well the longitudinal motion of the steel ball.
The results indicate to what extent the energy transfer to the diamonds depends on the test parameters and give us insight in the physics behind it.
The simulation has also allowed us to develop a test aimed at measuring the Fatigue Resistance of diamond grits. Fatigue failure is considered as an important failure mode for diamond grits in tools. Moreover, our measurements show that Fatigue Strength - and Impact Strength rankings of commercial grit types need not to be the same, but this is another story for a future conference.

From a purely theoretical point of view, the model is also a nice example of a simple deterministic system (a steel ball shaken in a cylinder) which shows, in some regions of parameter space, a chaotic behaviour.

5. Bibliography

[1] : *FEPA standard for measuring the relative strengths of saw diamond grits*, Edition 1, Federation of European Producers of Abrasives, Paris
[2] : W.H. Press, S.A Teukolsky, W.T. Vetterling, B.P. Flannery, *Numerical Recipes in Fortran, The art of scientific computing,* Second Edition, Cambridge University Press